Optimization of a window frame by BEM and genetic algorithm

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Abstract *Genetic algorithms and boundary elements have been used to find an optimal design of a plastic window frame with air chambers and steel stiffeners. The objective function has been defined as minimum heat loss subject to a constraint of prescribed stiffness and weight of the steel insert.*

1. Introduction

1.1 Algorithms of shape optimization

Optimization of engineering objects is an inherent portion of the design process. Intuition and experience have been the only available techniques for performing this task for generations of engineers. Introduction of computer techniques opened the possibility of using a systematic approach to optimization. The iterative algorithms used in this process require the solution of a sequence of boundary value problems, typically in domains of varying geometry. As such, computations are numerically very intensive, and nontrivial optimization problems were beyond the reach of practicing engineers for a long time.

The potential economic gains of shape optimization attracted many researchers to this problem (Fox, 1971; Gallagher and Zienkiewicz, 1973; Haftka *et al.*, 1990). An important theoretical tool developed to deal with shape optimization is the sensitivity analysis. The outcome of this technique is a set of sensitivity coefficients defining the influence of the increments of the design parameters onto the variation of the objective function. This set, the gradient of the objective function, is instrumental in many optimization algorithms (conjugate gradient, variable metric, etc.) whose outcome is the optimal shape of the domain under consideration. Various aspects of the sensitivity analysis in the context of shape optimization and inverse analysis have been widely discussed in the literature. The first monograph on this subject seems to be

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the book by Haung *et al.* (1986). Dems and Mróz (1998) present a state-of-the-art of sensitivity analysis in elasticity and thermoelasticity, and gives a comprehensive literature review of this topic.

The practical application of this technique is often cumbersome due to its mathematical complexity and inherent limitations. The latter situation results from the required properties of the objective function, which should be regular and should possess a positive definite Hessian. As a result, the case of discrete design parameters, specifically the variations in the topology of the domain (e.g. introduction of openings), is not straightforward. Another disadvantage of the standard optimization techniques is their tendency to stall at local optima of the objective function.

Genetic algorithms, whose principle mimics the natural selection process, offer an elegant way of circumventing these disadvantages. The algorithms do not require the calculation of the sensitivity coefficients and can readily be employed to problems with varying topology. Another advantage of genetic algorithms is their robustness in the presence of local optima. On the other hand, the computing time of genetic algorithms is much longer than the case of standard nonlinear programming. The recent reduction in computing costs along with the parallel computing options have made genetic algorithms competitive with standard optimization techniques.

Genetic algorithms (often referred to as evolutionary computations) have been introduced independently by two groups of researchers working in the USA (Fogel *et al.*, 1966; Holland, 1975) and one in Germany (Rechenberg, 1973). The monograph (Goldberg, 1989) presented an unified approach to the problem and is the most frequently cited book in genetic algorithms. Recently, a monograph on applications of evolutionary algorithms has been published in Poland (Arabas, 2001). The important question of parallelization of the genetic calculations is discussed in a review (Seredyński, 1998).

The evaluation of the objective function in the case of shape optimization is achieved by the solution of a boundary value problem in a region of complex shape. In nontrivial cases, this can be accomplished only by using the numerical techniques. This in turn requires the generation of a numerical grid. The finite element method, a domain discretization technique, entails a generation of the grid throughout the entire computational domain. This task, although conceptually trivial, is computationally fairly demanding.

Using the boundary element method (BEM), instead of the FEM, offers a significant advantage, as the discretization of the domain in most cases is restricted solely to the boundary. Thus, due to the reduction of the dimensionality, the automatic grid generation in BEM is much easier to implement than in FEM. Therefore, if the problem at hand can be reduced to a boundary only formulation, BEM is a preferred numerical technique in shape optimization.

Summing up this short review of the available shape optimization techniques, the combination of genetic algorithms and the BEM seem to be the most attractive technique for solving this class of problems, and this has been recognized by Kita and Tani (1997). A recent paper of Burczyński *et al.* (2002) discusses the application of BEM and evolutionary algorithms in optimization and identification.

1.2 Window frame optimization

The increasing energy costs and lower admissible CO_2 emission are the driving forces for the need to reduce heat losses from buildings. The building envelope elements exert a major influence on the energy consumption of buildings. In the early stage of the R&D process in this field, the main stress has been on increasing the thermal resistance of the walls. Progress in this area has been achieved mainly by the introduction of new materials and additional layers of thermal insulation. Because of the new regulations in national and international standards, the admissible value of the heat losses of the walls has been considerably reduced in the last few decades.

Another potential source for the reduction in heat losses from buildings is the optimization of the ventilation system. Research in this area concentrates on decreasing the amount of infiltrating air and introducing forced ventilation equipped with recuperating heat exchangers.

However, about 30 per cent of heat is lost through the windows in a building. Typical windows consist of double glazed panes and wooden, plastic or metal frames. Many efforts have therefore been made to reduce the transmissivity of the glazing system. The heat resistance of a double pane can be increased by selecting an optimal distance between the glass sheets and filling this gap with a low conductivity gas. Radiative heat losses through the glazing system are reduced by the introduction of thin coatings and by using glass of low emissivity. In contemporary designs, the total heat losses from panes are as low as $1.1\,\mathrm{W/m^2K}$. At the current energy price level, further insulation improvement does not seem to be economically justified.

Window frames have smaller surface area than window panes, thus, for a long time, the optimal thermal design of these elements has been of secondary importance. At current levels of glazing and wall insulation, the question of heat losses from window frames has become more important.

The present paper deals with the optimal design of a plastic window frame. This kind of frame has become very popular due to its low price, easy maintenance and reasonable insulation properties. To increase the thermal resistance of the frame and minimize its weight, the air cavities are introduced. However, as the plastic frames do not have the required stiffness, metal profiles are inserted in the frame and the presence of a high conducting metal increases the heat losses. The topic of the present study is the optimal placement of the stiffener and the air cavities in order to achieve minimum heat losses through the frame while maintaining the required stiffness and using the same amount of metal.

2. Formulation of the problem

2.1 Heat transfer

A 2D steady-state heat transfer problem is considered. The frame consists of three materials: PVC, air and steel. Constant material properties have been assumed. The values taken in the calculations are shown in Table I. For the temperature differences and geometrical dimensions occurring in the problem, both natural convection and radiation are of minor importance in the air filled enclosures. Thus, it is assumed that the heat in the cavities is transferred solely by conduction.

Prescribed boundary conditions are shown in Figure 1. On the portions of the contour exposed to the environment and in contact with the air in the room, Robin boundary conditions are prescribed. The values of the indoor and outdoor temperatures were set to +20 and -20°C, which is in agreement with the Polish standards PN-82/B-02402 and PN-82/B-02403. The values of the indoor and outdoor heat transfer coefficients, 23 and 8 W/m^{2}K, have been taken from another Polish standard PN-EN ISO 6946. Heat transfer through the remaining portions of the external surface of the frame has been neglected. On the interfaces between the different materials, ideal thermal contact,

Table I.
Material properties used in the calculations

Material	Heat conductivity (W/m^{2}K)
PVC	0.163
Air	0.023
Steel	58.00

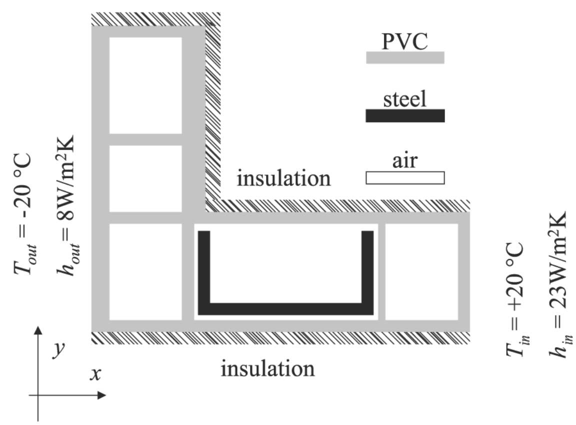

Figure 1.
Geometry and prescribed boundary conditions for the window frame

i.e. continuity of both temperature and heat flux has been assumed. The geometry of the numerical examples investigated is a simplified version of a real frame taken from Technical approval ITB (1998).

2.2 Formulation of the optimization problem

The objective of the optimization is to minimize the heat losses subjected to several constraints.

It is assumed that the element of the frame can be modeled as a beam. Additional stiffness resulting from the connections with other elements of the frame is neglected, which is a conservative assumption. The standard 1D beam equation used in the study is given by

$$EI\,\frac{\mathrm{d}^4 u}{\mathrm{d}x^4} = 0 \tag{1}$$

where u is the deflection of the axis of the beam, E and I are the Young's modulus and moment of inertia, respectively.

As the contribution of the plastic to the overall stiffness of the frame is negligible, the measure of the stiffness is the moment of inertia of the metal insert with respect to the vertical (y) axis passing through the centre of gravity.

With this definition of stiffness, the following additional conditions should be fulfilled:

- minimum stiffness should be maintained,
- amount of metal should not exceed a prescribed value,
- stiffener is contained within air cavities (and not immersed in plastic),
- outer contour of the plastic frame is not changed by the algorithm,
- thickness of the plastic interior walls is 1 mm while that of the exterior is 3 mm, and
- geometry of the frame is approximated by a set of line segments.

The design variables are contractions, expansions and translations of the air cavities, and deformations of the steel insert. The location of the *characteristic points* of the boundary, i.e. the corner points of the air cavities and the stiffener, is expressed in terms of decision variables defined as the coordinates of some *control points*. In the developed algorithm, the coordinates of the characteristic points are defined as an arbitrary linear combination of the coordinates of the control points. This approach offers significant flexibility in defining the admissible variation of the geometry.

3. Numerical technique

3.1 Solution of the heat conduction problem

The heat losses from the frame have been computed using BETTI, a boundary element code (Białecki and Kuhn, 1993). The details of the BEM technique are available in Wrobel (2002). Only the basic steps of BEM are mentioned in the present paper.

The first step in the BEM is a transformation of the original boundary value problem in a homogeneous domain into an equivalent integral equation of the form (Wrobel, 2002)

$$c(\mathbf{p})T(\mathbf{p}) = \int_C [q(\mathbf{r})T^*(\mathbf{p},\mathbf{r}) - T(\mathbf{r})q^*(\mathbf{p},\mathbf{r})]\,\mathrm{d}C(\mathbf{r}) \tag{2}$$

where $\mathbf{r}$ and $\mathbf{p}$ are vector coordinates of the current and observation points, respectively. T is the temperature and q the associated heat flux $q = -k\nabla T\cdot\mathbf{n}$, where k is the heat conductivity and $\mathbf{n}$ is the outward unit normal vector of the contour, T^* is the fundamental solution of the Laplace equation and $q^* = -k\nabla T^*\cdot\mathbf{n}$. $c(\mathbf{p})$ is a fraction of the angle with vertex at $\mathbf{p}$ subtended in the domain.

The next step is the discretization of equation (2). The first stage of this procedure is the subdivision of the contour into a set of (boundary) elements. The geometry of every element is approximated using locally based shape functions, expressed in local coordinates. The same set of functions is used to approximate the variation of temperature and normal flux within elements. Introduction of these approximations into the original integral equation (2) produces residuals. The final set of equations is then generated by the nodal collocation, i.e. requiring that the residuals vanish a set of nodal points. The result reads

$$\mathbf{H}^i\mathbf{T}^i + \mathbf{G}^i\mathbf{q}^i = 0 \tag{3}$$

where $\mathbf{H}$ and $\mathbf{G}$ are the influence matrices and the vectors $\mathbf{T}$ and $\mathbf{q}$ are the values of temperature and heat fluxes at the boundary nodes. Superscript i refers to the subregion number.

The procedure is repeated in all subregions and the sets of linear equations corresponding to the subregions are linked by enforcing the continuity of temperature and heat flux on the interface between the adjacent subregions.

In the present study, the geometry as well as the distributions of both boundary temperature and heat flux have been approximated by isoparametric continuous quadratic elements. In the presence of corner points at the interface, this type of element fails to produce the sufficient number of equations (Białecki *et al.*, 1993). To circumvent this problem, a pair of constant elements meeting at such points have been introduced.

3.2 Constraints

To check the satisfaction of the constraints, evaluation of the surface area, coordinates of the mass centre and the moment of inertia are required. All these quantities may be expressed in terms of the surface integrals, namely

$$A = \int_A \mathrm{d}A \tag{4}$$

$$x_0 = \frac{\int_A x \, \mathrm{d}A}{A} \tag{5}$$

$$I_{yy} = \int_A (x - x_0)^2 \, \mathrm{d}A \tag{6}$$

where A is the surface area, x_0 is the x coordinate of the mass centre and I_{yy} is the moment of inertia with respect to the y axis passing through the mass centre.

The evaluation of these surface integrals can be significantly simplified by converting them into the contour integrals. This has been accomplished by making use of the Stokes theorem

$$\oint_C \vec{w} \cdot \mathrm{d}\vec{C} = \int_A curl\vec{w} \cdot \mathrm{d}\vec{A} \tag{7}$$

where $\vec{w}$ is an arbitrary vector and C is the contour of the surface A.

As the surface of integration lies in the xy plane, the normal infinitesimal surface vector is defined as $\mathrm{d}\vec{A} = [0, 0, \mathrm{d}x\mathrm{d}y]$ and the tangential contour line vector has the form of $\mathrm{d}\vec{C} = [\mathrm{d}x, \mathrm{d}y, 0]$

Denoting the vectors used to calculate the surface area, center of gravity and moment of inertia by w_A, w_y and w_I, respectively, their curls are defined as

$$\mathrm{curl}\vec{w}_A = [0, 0, 1] \tag{8}$$

$$\mathrm{curl}\vec{w}_y = [0, 0, x] \tag{9}$$

$$\mathrm{curl}\vec{w}_I = [0, 0, (x - x_0)^2] \tag{10}$$

It can be readily proved that the vectors $\vec{w}$ should be defined as

$$\vec{w}_A = [0, x, 0] \tag{11}$$

$$\vec{w}_y = [-xy, 0, 0] \tag{12}$$

$$\vec{w}_I = [-y(x - x_0)^2, 0, 0] \tag{13}$$

The parametric equations of the line segments constituting the contour of the frames can be written as

$$x = x_b + (x_e - x_b)t \tag{14}$$

$$y = y_b + (y_e - y_b)t \tag{15}$$

where the indices b and e correspond to the start and end points of the segments, respectively, and t represents a parameter assuming values in the interval $[0, 1]$. Using the parametric representations (14) and (15), the infinitesimal tangential contour vector can be expressed as

$$\mathrm{d}\vec{C} = [x_e - x_b, y_e - y_b, 0]\,\mathrm{d}t \tag{16}$$

Using equations (7-16), the surface area, coordinates of the mass center and the moment of inertia can be written as a sum of definite integrals over $[0, 1]$ intervals corresponding to the subsequent line segments constituting the contour of the frame.

3.3 Genetic algorithm

The evaluation of the optimal geometry of the frame, in the sense of minimum heat losses subject to the constraints defined in the previous section, has been accomplished using a standard genetic algorithm. The details of this technique have been described in Goldberg (1989).

The main features of the implemented version of the algorithm are given in the following description.

The procedure starts with the creation of an initial population consisting of N_G identical members. The fitness function used is expressed in terms of heat losses Q_L by relationship $fitness = (Q_L)^{-p}$, where p is a user defined constant.

In the subsequent steps of the procedure, new generations are created. The number of individuals in a generation does not change throughout the iterative process and the new generation is generated in three stages: selection, mutation and mating.

The probability of selecting candidates for the next generation is proportional to their fitness functions. The genes of the selected members undergo creeping mutation and the probability of this process is P_{m}. If after this operation the genes of the member fulfill the prescribed constraints, then the individual is included in the new generation, otherwise, the procedure of generating a new member is repeated.

Mating starts with the random selection of two members of the new population. The probability of selection is the same for all members. After a pair is selected, the crossover is triggered with a probability of P_{c}. In the process of procreation, the location of the chromosome interchange is selected at random. If the offspring fulfill the constraints, then they substitute the parents, otherwise, the parents remain in the population. The number of individuals selected for crossover is equal to the number of individuals in the generation. The version of the genetic algorithm used in this work uses the predefined number N_{p} of generations that have been created as the stopping criterion. The termination condition can also be formulated in terms of the convergence defined as the improvement of the fitness in the best member of the subsequent generations.

The coordinates of the control points are coded as genes associated with a given member of the population. Gen is coded as a sequence of 32 bits. The smallest change of the displacement within the procedure is defined as 0.001 mm. This is much higher than the accuracy of frame manufacturing. From the practical point of view, the changes of the geometry can therefore be treated as continuous. The number of genes in a chromosome is equal to the number of degrees of freedom, i.e. admissible displacements of the control points.

4. Numerical examples

Even in the very simplified geometry considered in this paper, the number of design parameters is very large. The present study is an introductory step to the optimization of a movable and fixed window framework taking into account their thermal interaction with the glass pane and the wall. The aim of the numerical examples discussed in this paper is to identify the crucial degrees of freedom whose change would significantly influence the objective function. Another purpose of this paper is to tune the genetic algorithm by finding out the values of its characteristic parameters controlling the convergence of the procedure. Because of the required CPU times, this kind of parametric study would be difficult to perform in the case of the target being a large computational domain.

4.1 Example 1

In this example, the initial moment of inertia of the metal insert has been chosen as 2.12 cm^4, i.e. it was larger than the minimum required value of 1.3 cm^4. The motivation for such a choice of the starting solution was to check whether the procedure will reduce, as the common sense suggests, the moment of inertia to the predefined minimum. The stiffener has been allowed to bend in the center of its segments. The surface area of the insert was constant throughout the optimization process, namely 1.17 cm^2. The design parameters used in this example are shown in Figure 2.

This example has been used to study the influence of the control parameters of the genetic algorithm on the convergence and numerical efficiency.

The efficiency and accuracy of the genetic algorithm depends on the values of a set of tuning parameters:

- number of individuals in the generation, N_g;
- probability of mutation, P_m;
- probability of crossover, P_c;
- power used in the definition of the fitness function, p;
- number of populations, N_p.

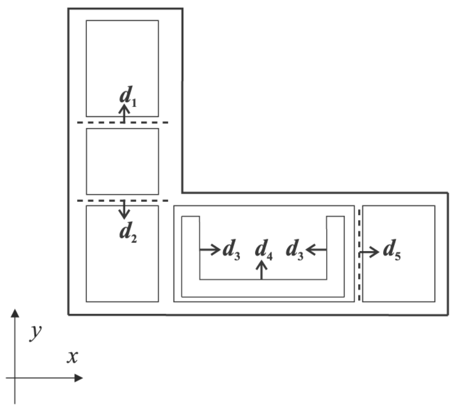

Figure 2.
Design parameters used – example 1

Only the last quantity can be adjusted during the computations by a simple check of the convergence. There is no sound theory on how the remaining parameters should be selected. Thus, it is a common practice to choose these values using heuristic reasoning. To gain some experience on how these parameters influence the convergence rate, several test runs have been made. The best set of parameters have been used in the next numerical examples.

The methodology used in these tests is simple: while keeping the values of all but one parameter at the same level, the parameter of interest was changed. The standard values of the parameters used in these tests were as follows: number of individuals in the generation $N_{\mathrm{g}} = 30$, number of populations $N_{\mathrm{p}} = 100$, probability of mutation $P_{\mathrm{m}} = 0.15$, probability of crossover $P_{\mathrm{c}} = 0.5$ and power of the fitness function $p = 1$.

In the first set of calculations, the power used in the definition of the fitness function has been examined. The selected values were $p = 0.3$, 1 and 3. As can be seen in Figure 3, this parameter has practically no influence on both the convergence rate and the value of the optimum.

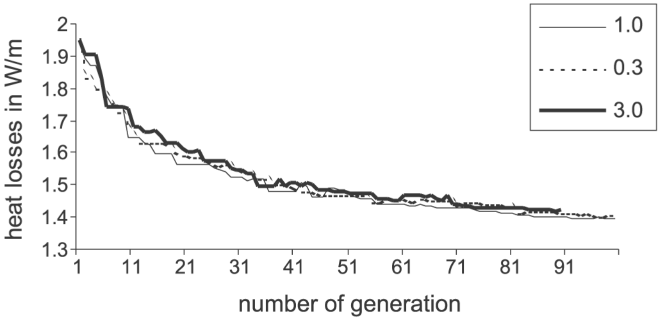

Figure 3.
Plots of the lowest heat losses with a given population showing the influence of the power used in the fitness function. The parameter on the curves are the values of p in the definition fitness $= 1/Q_L^p$

In the second set of calculations, the number of individuals in the generation has been varied. The values used in calculations were $N_g = 10$, 20 and 30. Figure 4 shows that for $N_g = 10$, the convergence rate is much lower than the other values. Populations of 20 and 30 individuals produce almost the same results.

Similar tests have been conducted out for different values of the probability of mutation. Here, values of $P_m = 0.05$, 0.15 and 0.45 have been selected. The results are shown in Figure 5. While $P_m = 0.05$ result in a slow convergence, probabilities $P_m = 0.15$ and 0.45 give practically the same results.

The final result of the optimization was a reduction in the heat losses from 1.94 to 1.38 W/m, i.e. about 30 per cent. At the optimal point, the moment of inertia has, as expected, reached the lowest admissible value of 1.3 cm^4. These results have been obtained taking 100 generations with population of one generation equal to 30 and the probabilities of mutation $P_m = 0.15$ and mating $P_c = 0.5$. Figure 6 shows the history of the reduction of the heat losses within the optimization process and Figures 7 and 8 show the initial and resulting geometries of the frame.

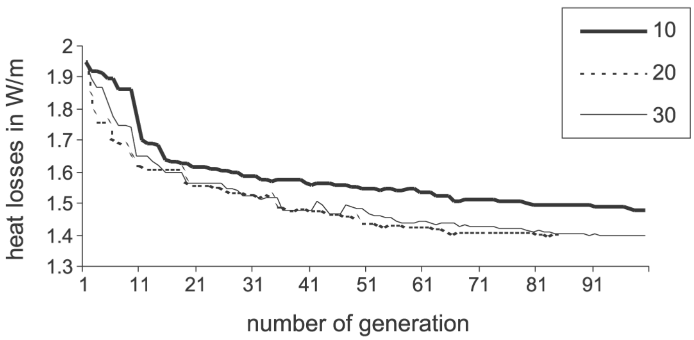

Figure 4.
Plots of the lowest heat losses within a given generation showing the influence of the number of individuals in the population

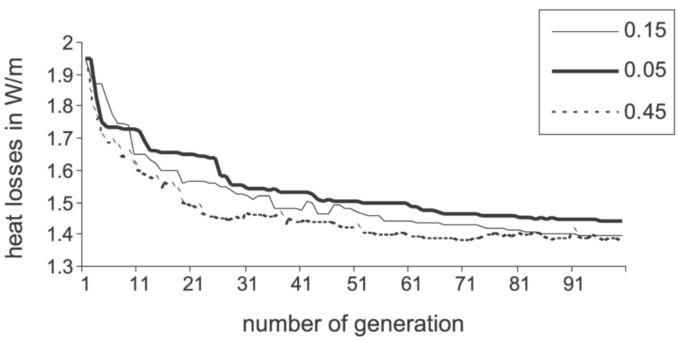

Figure 5.
Plots of the lowest heat losses within a given generation showing the influence of the probability of mutation

Another outcome of these tests was the observation that the optimum can be achieved for two different configurations of walls separating the three leftmost cavities. Thus, more than one optimal configurations of the frame may exist.

4.2 Example 2

In this example, the starting configuration was the same as in the previous example. The moment of inertia has been kept constant at the level of $2.12\,\mathrm{cm}^4$.

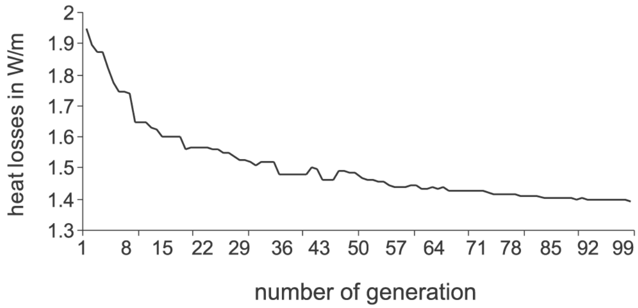

Figure 6.
Plots of the lowest heat losses within a given generation showing the reduction of heat losses in the course of iterations

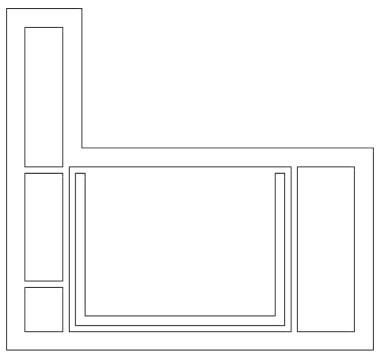

Figure 7.
Starting configuration of the frame – example 1

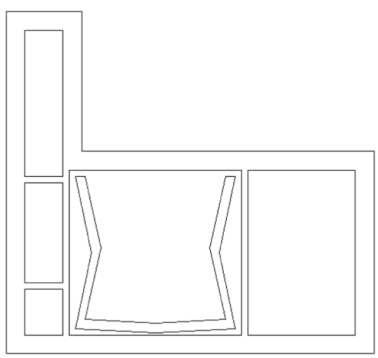

Figure 8.
Resulting geometry of the frame after optimization – example 1

A constant value for the surface area has been taken as in the previous example, namely 1.17 cm^2. The thickness and length of the horizontal and vertical arms of the stiffener were allowed to change independently and the initial value of the thickness was 1.5 mm. The lowest admissible thickness was set to 1 mm. This condition was introduced to prevent solutions with too slender profiles. The angle of inclination of the vertical arms were allowed to vary. The surface area of the insert was constant. The remaining parameters of the genetic algorithm were taken as in the previous example. A sketch of the degrees of freedom is shown in Figure 9.

The result of the optimization was a reduction in the heat losses from 1.94 to 1.72 W/m, i.e. about 13 per cent. These results have been obtained by taking 250 generations. Figures 10 and 11 show the initial and resulting geometries of the frame. The optimal values of the thicknesses were $d_7 = 2.43$ mm, $d_8 = 2.27$ mm and $d_9 = 1$ mm. It should be noted that the latter value is the lowest admissible thickness of the profile.

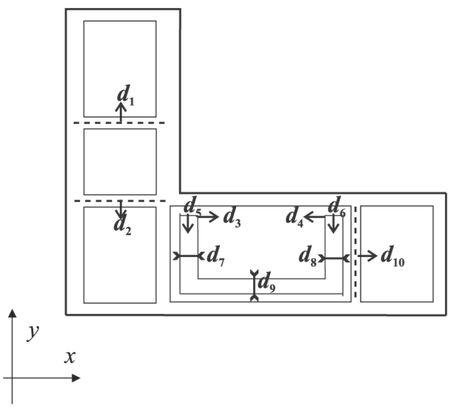

Figure 9.
Design parameters used – example 2

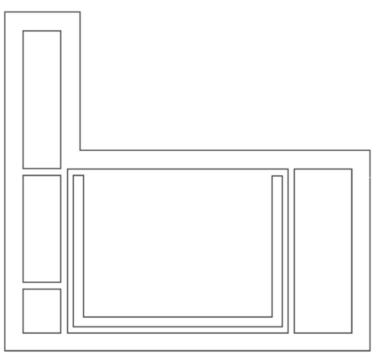

Figure 10.
Starting configuration of the frame – example 2

4.3 Example 3

In this example, the initial moment of inertia has been kept constant at the level of the admissible value, i.e. 1.3 cm^4. The thickness and length of the horizontal and vertical arms of the stiffener were allowed to change. While the thickness of the left and right arm were the same, their lengths could vary independently. No constraint has been imposed on the minimum thickness of the profile and the surface area of the insert was constant. The remaining parameters of the genetic algorithm were taken as in the previous example. A sketch of the degrees of freedom is shown in Figure 12.

The final result of the optimization was a reduction in the heat losses from 1.66 to 1.44 W/m, i.e. about 12 per cent. These results have been obtained by taking 450 generations. Figures 13 and 14 show the initial and resulting geometries of the frame. The obtained thickness of the vertical arms was $d_5 = 3.28$ mm while the thickness of the horizontal arm was $d_6 = 0.87$ mm.

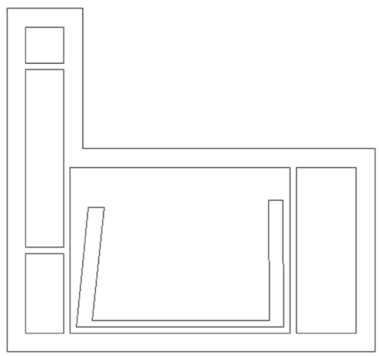

Figure 11.
Resulting geometry of the frame after optimization – example 2

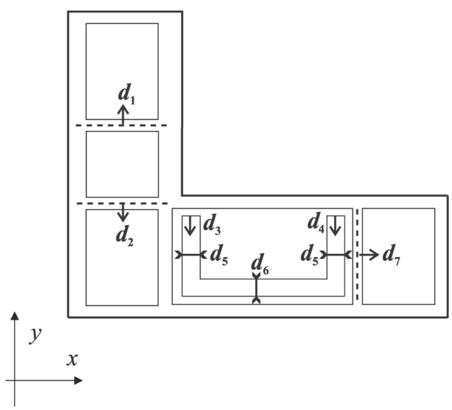

Figure 12.
Design parameters used in example 3

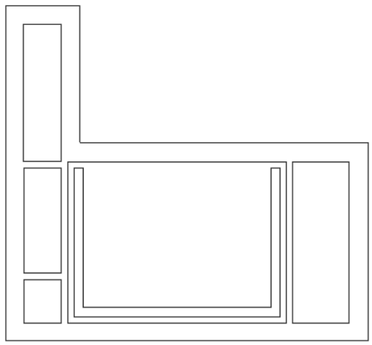

Figure 13. Starting configuration of the frame – example 3

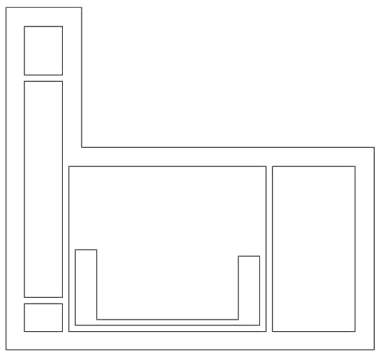

Figure 14. Resulting geometry of the frame after optimization – example 3

5. Conclusions

The application of genetic algorithms with fitness calculated by the BEM has proved to be a robust technique in dealing with the shape optimization problem, where heat transfer and elasticity interact. The calculations carried out show the possibility of a substantial reduction in the heat losses from a window frame. This can be achieved by a simple modification of the geometry of the plastic frame and the steel stiffener. In the final configuration, the heat losses may be reduced by as much as 30 per cent. The heat losses can be reduced by decreasing the length of the horizontal arm of the stiffener and its thickness, while increasing the thickness of the vertical arms and changing their inclination and shape.

Test runs have given some optimal values of the tuning parameters of the algorithm. This knowledge and the observations concerning the possible degrees of freedom will be used in the next stage of the research, when more complex configurations of the computational domain will be considered.

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